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A CLASS OF PLANE SELF-SIMILAR MOTIONS OF A NON-NEWTONIAN FLUID WITH NONLINEAR THERMOPHYSICAL PROPERTIES

O. N. Shablovskii

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The plane self-similar solution of a set of complete equations for the dynamics of a nonlinearly viscous fluid and the energy equation is obtained analytically with the temperature dependence of the transfer coefficients taken into account.

1. INITIAL EQUATIONS AND NEW INDEPENDENT VARIABLES

We take the generalized Z. P. Shul'man model of a nonlinearly viscoplastic incompressible fluid as a basis and we write the equations of two-dimensional plane nonstationary motion, the continuity and heat balance equations [1]:

$$u_t + (p/\rho + u^2 - \tau_{11}/\rho)_x + (uv - \tau_{12}/\rho)_y = 0, \tag{1}$$

$$v_t + (uv - \tau_{12}/\rho)_x + (p/\rho + v^2 - \tau_{22}/\rho)_y = 0, \tag{2}$$

$$u_x + v_y = 0, \quad \rho \equiv \text{const}, \tag{3}$$

$$\rho c_p (T_t + uT_x + vT_y) = (\lambda T_x)_x + (\lambda T_y)_y + A^2 B, \tag{4}$$

$$\tau_{11} = 2Bu_x, \quad \tau_{12} = \tau_{21} = B(u_y + v_x), \quad \tau_{22} = 2Bv_y, \tag{5}$$

$$A = [2u_x^2 + 2v_y^2 + (u_y + v_x)^2]^{\frac{1}{2}}, \quad B = [\tau_0^{\frac{1}{n}} A^{-\frac{1}{m}} + \mu^{\frac{1}{m}}]^n A^{\frac{n}{m} - 1},$$

$$\mu = \mu(T), \quad \lambda = \lambda(T), \quad c_p = c_p(T).$$

We here assume p differentiable with respect to x, y, t and u, v, T twice differentiable with respect to x, y and once with respect to t. All these derivatives as well as the second mixed derivatives of the functions p, u, v, T in the arguments x, y, t are considered continuous in the space–time domain under consideration.

Equation (2) can be satisfied by taking

$$v = -\xi_y, \quad uv - \frac{\tau_{12}}{\rho} = \eta_y, \quad \frac{p - \tau_{22}}{\rho} + v^2 = \xi_t - \eta_x.$$

We substitute the expression p from the last formula into (1), regroup the terms therein by using the equality $\xi_{tx} = \xi_{xt}$ and satisfy the equation obtained as follows

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$$u + \xi_x = -D_y, \quad u^2 - v^2 - \eta_x + (\tau_{22} - \tau_{11})/\rho = C_y, \quad uv - \tau_{12}/\rho = D_t - C_x.$$

Here ξ, η, C, D are auxiliary unknown functions of the arguments x, y, t. We now use the notation $\xi_x \equiv \varphi = -u - D_y$, $\xi_y = -v$, $\xi_t \equiv \psi = \pi + v^2 + \eta_x - \tau_{22}/\rho$, $p/\rho = \pi + E(t)$, where E(t) is arbitrary, and we eliminate the function $\xi(x, y, t)$ from the number of dependent variables by using the condition of equality of the mixed second-order derivatives:

$$-v_x = \varphi_y, \quad -v_t = \psi_y, \quad \varphi_t = \psi_x.$$

Taking account of the initial assumption about the smoothness and differentiability of the functions characterizing the flow, we conclude that only two of these three equations are independent while the third is a result of their differentiation. We later consider the first two of these equations.

Using the equation $d\xi = \varphi dx - v dy + \psi dt$, we perform the mutually one-to-one passage from the x, y plane to the plane of the new independent variables x, ξ by the formula

$$y + \int_{\xi_0}^{\xi} \frac{d\xi}{v(\xi, x_0, t_0)} - \int_{x_0}^{x} \frac{\varphi}{v}\bigg|_{t_0} dx - \int_{t_0}^{t} \frac{\psi}{v}\, dt = \text{const}, \quad \frac{D(x, \xi)}{D(x, y)} = -v \neq 0$$

or equivalently

$$y + \int_{\xi_0}^{\xi} \frac{d\xi}{v(x, \xi, t)} = y_0(x, t), \quad v \neq 0, \ \xi_0 \equiv \text{const}, \tag{6}$$

where x_0, ξ_0, t_0 are the initial values of the appropriate arguments while $y_0(x, t)$ is connected with the flow parameters for $\xi = \xi_0$ by the relations

$$\frac{\partial y_0}{\partial x} = \frac{\varphi}{v}\bigg|_{\xi_0}, \quad \frac{\partial y_0}{\partial t} = \frac{\psi}{v}\bigg|_{\xi_0}.$$

The differentiation operations are transformed as follows:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'} + \varphi \frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial y} = -v \frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} + \psi \frac{\partial}{\partial \xi},$$
$$x = x', \quad t = t'. \tag{7}$$

After such a transformation, we obtain a system of seven equations for the functions p, u, v, T, η, C, D of the arguments x, ξ, t in place of (1)-(5). We omit writing these equations in the new variables.

The condition $v \neq 0$ for mutual one-to-oneness indicates that flows for which the fluid velocity component along the OY axis retains a constant sign for $t \geqslant t_0$ can be considered on the basis of this transformation.

The hydrodynamic meaning of the function $\xi(x, y, t)$ is that the nonpenetration condition is satisfied automatically along the line $\xi = \xi_0 \equiv \text{const}$ while the adhesion condition is satisfied for $D_\xi(x, \xi_0, t) = 0$.

2. SELF-SIMILAR VERSION OF THE TRANSFORMED MOTION EQUATIONS

We assume the fluid yield point constant $\tau_0 \equiv \text{const}$ and its thermophysical parameters dependent on the temperature according to a power law

$$c_p = c_0 T^{-1}, \ \mu = \mu_0 T^{\frac{1}{\mu_1}}, \quad \lambda = \lambda_0 T^{\frac{1}{\mu_1} - 1}, \quad \mu_1 \neq 0, \ c_0, \ \mu_0, \ \lambda_0 - \text{const},$$
$$T \in [T', T'']. \tag{8}$$

We furthermore assume a self-similar flow characterized by the dependences

$$u = u(\alpha, \beta), \quad v = v(\alpha, \beta), \quad \pi = \pi(\alpha, \beta), \quad T = (t + l^2)\,\theta(\alpha, \beta), \tag{9}$$
$$D = (t + l^2) F(\alpha, \beta), \quad C = (t + l^2) G(\alpha, \beta), \quad \eta = (t + l^2) H(\alpha, \beta),$$

$$\alpha = \frac{x}{t + l^2}, \quad \beta = \frac{\xi}{t + l^2}, \quad l \equiv \text{const}, \quad l \neq 0. \tag{10}$$

The relationship between the self-similar variables α, β, γ is obtained by using (6):

$$\gamma \equiv \frac{y}{t+l^2} = -\int\limits_{\beta_*}^{\beta} \frac{d\beta}{v(\alpha, \beta)} + \gamma_*(\alpha), \quad \beta_* \equiv \mathrm{const}, \quad \frac{d\gamma_*}{d\alpha} = \left.\frac{\varphi}{v}\right|_{\beta_*}. \tag{11}$$

Equations (1)-(5) transformed to the independent variables x, ξ, t have the following form in the self-similar case (9)-(10):

$$\begin{gathered} v_\alpha + \varphi v_\beta = v\varphi_\beta, \quad \psi v_\beta - \alpha v_\alpha - \beta v_\beta = v\psi_\beta, \\ u_\alpha + \varphi u_\beta = v v_\beta, \quad \rho v(u + H_\beta) = \tilde{B}(v_\alpha + \varphi v_\beta - v u_\beta), \\ u^2 - v^2 + vG_\beta - \frac{4}{\rho}\tilde{B} v v_\beta = H_\alpha + \varphi H_\beta, \\ vH_\beta + F - \alpha F_\alpha + F_\beta(\psi - \beta) = G_\alpha + \varphi G_\beta, \end{gathered} \tag{12}$$

$$c_0\rho\theta^{1-\frac{1}{\mu_1}}[\mu_1\theta - \alpha\theta_\alpha + (\psi - \beta)\theta_\beta + u(\theta_\alpha + \varphi\theta_\beta) - v^2\theta_\beta] = \lambda_0\theta(\theta_\alpha + \varphi\theta_\beta)_\alpha + \lambda_0\left(\frac{1}{\mu_1} - 1\right)\theta_\alpha(\theta_\alpha + \varphi\theta_\beta) + \lambda_0\varphi\theta(\theta_\alpha +$$

$$+ \varphi\theta_\beta)_\beta + \lambda_0\left(\frac{1}{\mu_1} - 1\right)(\theta_\alpha + \varphi\theta_\beta)\varphi\theta_\beta + \lambda_0\left(\frac{1}{\mu_1} - 1\right)(v\theta_\beta)^2 + \lambda_0 v\theta(v\theta_\beta)_\beta + \theta^{2-\frac{1}{\mu_1}}(\mu_0\theta^{\frac{1}{\mu_1}})^{\frac{n}{m}}\tilde{A}^{\frac{n}{m}+1},$$

$$\begin{gathered} \varphi = vF_\beta - u, \quad \psi = \pi + v^2 + H_\alpha + \varphi H_\beta + \frac{2}{\rho}\tilde{B} v v_\beta, \quad A = \tilde{A}(\alpha, \beta)(t + l^2)^{-1}, \\ B = \tilde{B}(\alpha, \beta)(t + l^2). \end{gathered}$$

Therefore, one self-similar version is indicated here for the complete equations of motion and energy (1)-(5). A multilateral analysis of the self-similar motion regimes in boundary layers of rheologically complex fluids is given in [1]. The results of mathematical investigations of problems of nonlinearly viscous media motion are elucidated in [2].

3. EXAMPLE OF SELF-SIMILAR MOTION OF A NONLINEAR VISCOUS FLUID

We use (12) to describe the flow of a nonlinearly viscous fluid ($\tau_0 = 0$) by taking the ratio of the nonlinearity parameters (flow index) n/m = 2 and considering $2\mu_1 = -1$:

$$\begin{gathered} c_p = c_0 T^{-1}, \quad \mu = \mu_0 T^{-2}, \quad \lambda = \lambda_0 T^{-3}, \quad \tau_0 = 0, \quad c_0, \ \mu_0, \ \lambda_0 - \mathrm{const}, \\ T \in [T', T'']. \end{gathered} \tag{13}$$

We construct the solution of (12) for a fluid possessing the properties (13) in the form of the functional series

$$\begin{gathered} u = u_\delta(\alpha)\beta^\delta, \quad v = v_2\beta^2 + v_{\delta+1}(\alpha)\beta^{\delta+1}, \quad v_2 \equiv \mathrm{const}, \ v_2 \neq 0, \ v_3 \equiv 0, \\ \pi = 2\beta + \pi_2(\alpha)\beta^2 + \pi_\delta(\alpha)\beta^\delta, \quad \theta = \theta_1\beta + \theta_{\delta-1}(\alpha)\beta^{\delta-1}, \quad \theta_1 = \mathrm{const}, \ \theta_1 \neq 0, \\ F = F_{\delta-1}(\alpha)\beta^{\delta-1}, \quad G = G_{\delta-1}(\alpha)\beta^{\delta-1}, \quad H = H_{\delta-1}(\alpha)\beta^{\delta-1}, \\ \delta = 3, 4, 5, \ldots, \infty. \end{gathered} \tag{14}$$

Summation here is over the repeated subscript δ. In these expansions the v_2, θ_1 are arbitrary nonzero constants, and $\pi_2(\alpha)$ is an arbitrary function. The coefficients of the series are calculated by using the linear recursion relations

$$\dot{u}_3 = v_2^2, \quad \dot{F}_2 = -\frac{5}{3}v_2, \quad \dot{G}_2 = 3F_2 - \alpha\dot{F}_2, \quad \ddot{\theta}_2 = 0, \tag{15}$$

$$\dot{u}_4 = 0, \quad \dot{v}_4 = v_2(2F_2v_2 - u_3), \quad \rho H_2 = 3b_{-1}(F_2v_2 - u_3), \quad b_{-1}\theta_1^4 = \mu_0^2 v_2^2\sqrt{10}.$$

The next step in the calculations is

$$\begin{gathered} \dot{u}_5 = 6v_2v_4 + 3u_3^2 - 6F_2v_2u_3, \quad \dot{v}_5 = 6F_3v_2^2 - 2v_2u_4, \\ 4v_2b_{-1}\dot{F}_3 = 2G_2v_2\rho - 8b_0v_2^2 - \frac{1}{3v_2}[b_0(5v_2^2F_2 - 6v_2u_3)]_\alpha, \\ \dot{G}_3 = 2H_2v_2 + 4F_3 - \alpha\dot{F}_3 + 2F_2\left(\pi_2 + \frac{4}{\rho}v_2^2 b_{-1} + \dot{H}_2\right), \end{gathered} \tag{16}$$

$$3\rho v_2 H_3 = 6b_0 v_2 (F_2 v_2 - u_3) + 4v_2 b_{-1} (3v_2 F_3 - 2u_4),$$

$$v_2 \pi_3 = 2v_4 - \alpha \dot{v}_4 - v_2 \left[\dot{H}_3 + \frac{4}{\rho} b_0 v_2^2 + 2H_2 (2F_2 v_2 - u_3) \right],$$

$$\ddot{\theta}_3 = \frac{c_0 \rho}{2\lambda_0} \theta_1^3 + \frac{3}{\theta_1} \dot{\theta}_2^2 + \frac{16}{3} v_2^2 \theta_1, \quad b_0 \theta_1^5 = -4\mu_0^2 v_2^2 \theta_2 \sqrt{10}.$$

The quantities b_{-1}, b_0 are determined by the expansion $\tilde{B} = b_{\delta-4}\beta^{\delta-4}$.

The general form of the formulas to compute the coefficients of the series (14) is

$$\dot{u}_{k+2} = (k+3) v_2 v_{k+1} - 3v_2 u_3 (k-1) F_{k-1} + R_{k+2}(\alpha), \tag{17}$$

$$\dot{v}_{k+1} = (k-1)(k-2) v_2^2 F_{k-1} - v_2 (k-2) u_k + L_{k+1}(\alpha),$$

$$\lambda_0 \theta_1 \ddot{\theta}_k = Q_k(\alpha), \tag{18}$$

$$G_{k-1} = k f_{k-1} \alpha + g_{k-1} + N_{k-1}(\alpha), \; F_k = \frac{\rho (k-1) \alpha}{2(k+1) b_{-1}} g_{k-1} + f_k + M_k(\alpha), \tag{19}$$

$$v_2 (k-1) \pi_{k+1} = P_{k+1}(\alpha) - \alpha \dot{v}_{k+2} + (k+2) v_{k+2}, \quad k \geqslant 3. \tag{20}$$

Expansion of the right sides in (17)-(20) is awkward, and not presented here. We just note that finding the coefficient H_{k-1} at each step is an intermediate link in the calculation, and after H_{k-1} is eliminated, algebraic expressions R_{k+2}, L_{k+1}, Q_k, N_{k-1}, M_k, P_{k+1}, $k \geqslant 3$ are obtained that are comprised of already known coefficients found in the previous stages of the calculations. The coefficients G_{k-1}, F_k are sought from first order ordinary differential equations and contain one arbitrary constant of integration g_{k-1}, f_k. The coefficients $\pi_{k+1}(\alpha)$ are determined by the finite algebraic expressions (20) in which the arbitrary function $\pi_2(\alpha)$ enters (it was taken equal to a constant $\pi_2 \equiv$const in the calculations performed). The integrals of the linear differential equations (17) contain arbitrary constants that we denote by a_{k+2}, b_{k+1}, respectively. According to (18), for each coefficient θ_{k-1} we have two constants of integration m_{k-1}, n_{k-1}. Examination of (15)-(17) shows that u_3, v_4 contain the arbitrary constants a_3, b_4, f_2, while the coefficients u_{k+1}, v_{k+2} contain four arbitrary constants a_{k+1}, b_{k+2}, f_k, g_{k-1}, $k \geqslant 3$. Therefore, the fluid velocity, pressure and temperature are represented by the dependences

$$u = (v_2^2 \alpha) \beta^3 + U_{\delta+2}(\alpha) \beta^{\delta+2} + a_\delta \beta^\delta, \tag{21}$$

$$v = v_2 \beta^2 + V_{\delta+1}(\alpha) \beta^{\delta+1} + b_{\delta+1} \beta^{\delta+1}, \quad v_2 \neq 0, \; v \neq 0,$$

$$\frac{p}{\rho} = E(t) + 2\beta + \pi_{\delta-1}(\alpha) \beta^{\delta-1},$$

$$(t + l^2) T = \theta_1 \beta + \Phi_\delta(\alpha) \beta^\delta + \alpha m_{\delta-1} \beta^{\delta-1} + n_{\delta-1} \beta^{\delta-1}, \; \theta_1 \neq 0, \; \delta \geqslant 3,$$

$$U_{k+2} = \int_0^\alpha \dot{u}_{k+2} d\alpha, \; V_{k+1} = \int_0^\alpha \dot{v}_{k+1} d\alpha, \; \Phi_k = \int_0^\alpha \left[\int_0^\xi \ddot{\theta}_k d\xi \right] d\alpha, \quad k \geqslant 3.$$

The functions U_{k+2}, V_{k+1}, Φ_k in this solution are homogeneous polynomials of the argument α, where U_{k+2} and V_{k+1} contain the integration constants f_k, g_{k-1} that are in (19). The polynomials U_{k+2}, F_k are of degree $2k - 3$, while the polynomials V_{k+1}, Φ_k, π_k are of degree $2k - 4$, $k \geqslant 3$.

Therefore, the series (21) characterize the self-similar hydrodynamic and thermal fields in a nonlinearly viscous fluid and determine the fluid velocity components to the accuracy of four arbitrary functions of the argument β, and the temperature to the accuracy of two arbitrary functions of the argument β.

In this class of solutions the influence of the convective terms of the heat balance equation appears in the computation of the temperature, starting with the coefficient $\theta_6(\alpha)$, while the influence of the viscous energy dissipation starts with $\theta_7(\alpha)$.

4. PROBLEM OF NONLINEARLY VISCOUS FLUID SPREADING THROUGH A DOMAIN WITH MOVING BOUNDARIES [3]

Let us give a physical interpretation of the solution found. We take the domain $[0, \alpha_1; \beta_0, \beta_1(\alpha)]$ which corresponds, in the plane of the self-similar variables α, γ, to domain $[0, \alpha_1; \gamma_0(\alpha), \gamma_1(\alpha)]$, where $0 < \alpha_1 < 1$, $0 < \beta_0 < \beta_1 < 1$.

Let us consider the problem of fluid motion in a domain with permeable moving boundaries, namely the case when satisfaction of the given conditions for the temperature and the velocity vector components on the two opposite boundaries $\alpha = 0$, $\alpha = \alpha_1$ is assured by the selection of the arbitrary functions contained in the solution (21). The two other boundaries $\gamma = \gamma_0(\alpha)$, $\gamma = \gamma_1(\alpha)$ are determined from the adhesion conditions, and the thermal hydrodynamic conditions thereon are dictated by the structure of the solution obtained.

The transformation of the independent variables (6) is realized for $v \neq 0$. The possibility of satisfying the adhesion condition on lines perpendicular to the OX axis is thereby eliminated. There is the same constraint in the self-similar case (11) relative to the coordinate α.

Let us consider a nonisothermal flow for which flow slip relative to the wall exists on the boundaries $x = x_i$, $i = 0, 1$ [1, 2]. We write the slip condition and the temperature jump in a form analogous to the corresponding conditions established in kinetic gas theory [4]:

$$x = x_i : u = u^i, \quad v = \zeta_i v_x + \omega_i T_y, \quad T = T_w^i + \tau_i T_x - \chi_i v_y, \quad i = 0, 1. \tag{22}$$

We here consider the fixed wall $x_0 = 0$ impermeable: $u^0 = 0$, and the fluid flows through the moving diaphragm $x = x_1 \equiv \alpha_1 (t + l^2)$, $\alpha_1 \equiv \text{const}$ at the velocity $u^1 \neq \alpha_1$; we assume the wall and diaphragm temperatures T_W^i, $i = 0, 1$ known.

In our self-similar case the conditions (22) have the same mode of writing:

$$\alpha = \alpha_i : v = \zeta_i (v_\alpha + \varphi v_\beta) - \omega_i v \theta_\beta, \tag{23}$$

$$\theta - \theta_w^i = \tau_i (\theta_\alpha + \varphi \theta_\beta) + \chi_i v v_\beta, \; \theta_w^i = \theta_{\delta-3}^i \beta^{\delta-3}, \quad i = 0, 1,$$

$$\alpha = \alpha_0 = 0 : u = 0, \quad \alpha = \alpha_1 : u = u^1(\beta), \tag{24}$$

where $\theta_W^i(\beta)$ are given analytic functions.

We do not here consider the question of an experimental determination of the dependences of the slip and temperature jump conditions on the flow parameters. We give these coefficients a priori so that they would possess an even dependence on the slip rate [2] and would satisfy the demands of self-similarity and of the boundary conditions (23) belonging to the class of solutions (21) for $E(t) \equiv 0$. The designated constraints will be satisfied, in particular, if we take

$$\zeta_i = T^{-1} (\zeta_{0i} p^{-1} + \zeta_{1i} v^2 p^{-5}), \quad \tau_i = T^{-1} (\tau_{0i} p^{-1} + \tau_{1i} v^2 p^{-5}), \tag{25}$$

$$\omega_i = T^{-2} (\omega_{0i} p^2 + \omega_{1i} v^2 p^{-2}), \quad \chi_i = \chi_{0i} + \chi_{1i} v^2 p^{-6},$$

where the quantities $\zeta_{ri}, \tau_{ri}, \omega_{ri}, \chi_{ri}, i = 0, 1, r = 0, 1$ are constants.

Conditions (23)-(25) permit determination of the arbitrary functions contained in the solution (21). The condition of no flow at $\alpha = 0$ yields $a_k = 0$, $k \geqslant 3$. Substituting the expansion (21) into condition (23) and grouping terms with identical powers of β we obtain linear algebraic equations for the coefficients θ_1, f_2, m_2 in the first stage of the calculations, where $\theta_1 \neq 0$ for $v_2 \neq 0$; we have four linear algebraic equations for the quantities g_2, f_3, m_3, n_2 in the second stage. Writing the mentioned equations and solving them are easily reproduced by assuming the formulas presented above for the expansion coefficients. For $k \geq 3$ a recursion sequence is obtained from the systems of equations for the unknown constants g_k, f_{k+1}, m_{k+1}, n_k (the writing in general form is omitted here). All the equations obtained are linear and have a unique solution for $v_2 \neq 0$.

By satisfying the four conditions (23) in this manner, we see that the slip rate $v(0, \beta) = v_2 \beta^2 + b_{\delta+1} \beta^{\delta+1}$ remains an arbitrary function related to the flow velocity $u^1 = u(\alpha_1, \beta)$ in a single-valued manner. The relation between the expansion coefficients for these functions in the argument β is easily traced by using (15)-(17).

We assume in the investigation of the convergence of the series (21) that the functions $b_{\delta+1}\beta^{\delta+1}$, $\theta_w^{(i)}(\beta)$, i = 0, 1 are analytic in the argument $\beta \in (0, 1)$. It can then be shown by the method of the majorant [5] that the fluid temperature on the wall $T^{(o)} = (\theta_1\beta + n_{\delta-1}\beta^{\delta-1})(t + l^2)^{-1}$ is also an analytic function in β.

Each of the series

$$U_{\delta+2}\beta^{\delta+2}, \quad V_{\delta+1}\beta^{\delta+1}, \quad \Phi_\delta\beta^\delta, \quad \alpha m_{\delta-1}\beta^{\delta-1} \tag{26}$$

satisfies the inequalities

$$|U_{k+2}(\alpha)| \leqslant \varepsilon_{k+2}, \ |V_{k+1}(\alpha)| \leqslant \varepsilon_{k+1}, \ |\Phi_k(\alpha)| \leqslant \varepsilon_k, \ |\alpha m_{k-1}| \leqslant \varepsilon_{k-1},$$
$$0 \leqslant \alpha < \alpha_{k-1}^0 < 1, \quad k \geqslant 3,$$

on the basis of the properties of homogeneous polynomials [6], and a majorizing number series $\varepsilon_{\delta-1}\beta^{\delta-1}$ can be constructed that converges for $\beta \in (0, 1)$ and there exists $\lim_{k\to\infty} \alpha_{k-1}^0 = \alpha^0 > 0$. This means that the series (26) converge uniformly for $\alpha \in [0, \alpha_2)$, $\beta \in (0, 1)$, where $0 < \alpha_1 \leqslant \alpha_2 \leqslant \alpha^0 < 1$.

We give the adhesion conditions on the other two permeable sections of the trapezoidal domain boundary $[0, \alpha_1; \beta_0, \beta_1]$, which permit finding the dependences

$$\gamma_j(\alpha) = \gamma_*(\alpha) - \int_{\beta^*}^{\beta_j(\alpha)} \frac{d\beta}{v(\alpha, \beta)}, \quad j = 0, 1, \ \gamma_*(\alpha) = \int_0^\alpha \left.\frac{\varphi}{v}\right|_{\beta_*} d\alpha,$$
$$\frac{d\beta_j}{d\alpha} = v(\alpha, \beta_j) F_\beta(\alpha, \beta_j) < \infty, \ \beta_j(0) = \beta_j^0, \ \beta_0^0 \neq \beta_1^0. \tag{27}$$

Here $\beta_* \in (0, 1)$ is a previously assigned number which denotes the lower boundary of the allowable values of the argument $\beta \in [\beta_*, 1)$.

The solution of (27) should be examined for values of $\alpha \in [0, \alpha_3)$, $0 < \alpha_1 \leqslant \alpha_3 < 1$, for which $0 < \beta_* \leqslant \beta_j(\alpha) < 1$, $\beta_0(\alpha) \neq \beta_1(\alpha)$. Such a number α_3 exists since the solution of the problem is analytic functions.

In order to assure satisfaction of the condition of mutual one-to-oneness $v \neq 0$ of the initial transformation of the independent variables, and the arbitrary function $v(0, \beta) = v_2\beta^2 + b_{\delta+1}\beta^{\delta+1}$ (the fluid slip rate over a fixed wall) must be given sign-definite for $\beta \in [\beta_*, 1)$. Then from the continuity of the function $v(\alpha, \beta)$ there follows the existence of the value $\alpha \in [0, \alpha_4)$, $0 < \alpha_1 \leqslant \alpha_4 < 1$, for which $v(\alpha, \beta) \neq 0$. As α_1 should be taken the least of the numbers α_2, α_3, α_4.

Since the interpretation of the solution with respect to the boundary conditions is realized by an inverse method for $\gamma = \gamma_j(\alpha)$, j = 0, 1, then the flow rate and temperature regime on these sections of the boundary are regulated by the structure of the solution (21) and are determined by the formulas

$$v_n(\alpha, \beta_j) = [(u^2 + v^2 - \alpha u - \gamma_j v)(u^2 + v^2)^{-\frac{1}{2}}]_{\beta=\beta_j}, \quad (t + l^2) T_j = \theta(\alpha, \beta_j).$$

The fluid pressure in this class of solutions is found to the accuracy of an arbitrary coefficient π_2 in the expansion (14).

NOTATION

x, y, Cartesian coordinates; t, time; α, β, γ, self-similar variables; u, v, projections of the velocity vector on the coordinate axes; p, pressure; T, temperature; ρ, fluid density; μ, effective viscosity coefficient; λ, heat-conduction coefficient; $c = \rho c_p$, specific heat of the fluid; ξ, new independent variable; η, D, C, auxiliary unknown functions; ζ, ω, τ, χ, slip and temperature jump coefficients. Subscripts: the independent variables as subscripts are partial differentiation; the dot above a function symbol is ordinary differentiation; δ is the summation subscript.

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CONVECTIVE HEATING OF A HALF-SPACE (NONSYMMETRIC CASE)

L. N. Germanovich and I. D. Kill'

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An exact solution of the heat-conduction problem for a half-space is obtained in a form convenient for calculations for an important case of a nonsymmetric temperature distribution of a medium bounding a half-space.

The solution of the heat-conduction boundary-value problem

$$\frac{\partial T}{\partial t} = \Delta T\,(r \geqslant 0,\ 0 \leqslant \varphi < 2\pi,\ z > 0,\ t > 0),\ T|_{t=0} = 0,$$
$$\frac{\partial T}{\partial z} = h\left[T - \exp\left(-\frac{r^2}{4\delta}\right)\sum_{n=0}^{\infty}\left(\vartheta_n \cos n\varphi + \tau_n \sin n\varphi\right)\right] \quad (z = 0,\ \tau_0 = 0) \tag{1}$$

written in dimensionless coordinates [1] is sought in the form

$$T = \sum_{n=0}^{\infty} [u_n(r,\ z,\ t)\cos n\varphi + v_n(r,\ z,\ t)\sin n\varphi] \quad (v_0(r,\ z,\ t) = 0). \tag{2}$$

Substituting (2) into (1) and following [1], we obtain

$$u_n = \vartheta_n \int_0^t I_n(r,\ \tau) f_0(z,\ \tau)\,d\tau, \quad v_n = \tau_n \int_0^t I_n(r,\ \tau) f_0(z,\ \tau)\,d\tau, \tag{3}$$
$$I_n = 2\int_0^{\infty} \lambda e^{-\lambda^2(1+\tau)} J_n(\lambda r)\,d\lambda,$$

where

$$f_0(z,\ \tau) = \frac{h}{\sqrt{\pi\tau}} \exp\left(-\frac{z^2}{4\tau}\right) - h^2 \exp(h^2\tau + hz)\operatorname{erfc}\frac{z + 2h\tau}{2\sqrt{\tau}}; \tag{4}$$

and $J_n(x)$ is the Bessel function. Using the technique of summation over gamma functions $\Gamma(x)$ [2] and the integral representation of Laguerre polynomials $L_k^n(x)$ [3], we find

$$I_n(r,\ \tau) = \frac{n}{2}(xy)^{\frac{n}{2}} y e^{-x} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{n}{2} + k\right)}{(n+k)!} y^k L_k^n(x), \quad x = \frac{r^2 y}{4},$$
$$y = \frac{1}{1+\tau}. \tag{5}$$

The case n = 0 corresponds to the axisymmetric problem (see [1]). For n = 2p (p = 1, 2, ...) we use the series for the generating function of the Laguerre polynomials [3]. We obtain

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